

My Research Visiting Card in Hamiltonian Graph Theory

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Abstract

We present eighteen exact analogs of six well-known fundamental Theorems (due to Dirac, Nash-Williams and Jung) in hamiltonian graph theory providing alternative compositions of graph invariants. In Theorems 1-3 we give three lower bounds for the length of a longest cycle C of a graph G in terms of minimum degree δ , connectivity κ and parameters \bar{p} , \bar{c} - the lengths of a longest path and longest cycle in $G \setminus C$, respectively. These bounds have no analogs in the area involving \bar{p} and \bar{c} as parameters. In Theorems 11 and 12 we give two Dirac-type results for generalized cycles including a number of fundamental results (concerning Hamilton and dominating cycles) as special cases. Connectivity invariant κ appears as a parameter in some fundamental results and in some their exact analogs (Theorems 3-10) in the following chronological order: 1972 (Chvátal and Erdős), 1981a (Nikoghosyan), 1981b (Nikoghosyan), 1985a (Nikoghosyan), 1985b (Nikoghosyan), 2000 (Nikoghosyan), 2005 (Lu, Liu, Tian), 2009 (Nikoghosyan), 2009a (Yamashita), 2009b (Yamashita), 2011a (Nikoghosyan), 2011b (Nikoghosyan).

Key words: Hamilton cycle, dominating cycle, longest cycle, generalized Hamilton cycles, circumference, hamiltonian graph theory, research visiting card.

1 The research area and the research objects

A cycle that contains every vertex of a graph is called a Hamilton cycle, and a graph that contains a Hamilton cycle is called hamiltonian. This terminology

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is used in honor of Sir William Rowan Hamilton, who, in 1865, described an idea for a game in a letter to a friend. This game, called the icosian game, consisted of a wooden dodecahedron with pegs inserted at each of the twenty vertices. These pegs supposedly represented the twenty most important cities of the time. The object of the game was to mark a route (following the edges of the dodecahedron) passing through each of the cities only once and finally returning to the initial city. Hamilton sold the idea for the game.

The classic hamiltonian problem; determining when a graph contains a Hamilton cycle, has long been fundamental in graph theory and has been intensively studied by numerous researchers. The problem of determining Hamiltonicity of a graph is one of the NP-complete problems that Karp listed in his seminal paper [6], and accordingly, one cannot hope for a simple classification of such graphs.

Today, any path or cycle problem, related to their lengths, is really a part of this general area. However, in the last 60 years much of research in hamiltonian graph theory has focused on the following two general problems:

- find sufficient conditions for Hamilton and dominating cycles and for their some generalizations,
- find lower bounds for the circumference - the length of a longest cycle in a graph.

Nowdays, we cannot even begin to completely survey the extensive literature on hamiltonian graph theory.

2 Notations and definitions

We consider only finite undirected graphs without loops or multiple edges. A good reference for any undefined terms is [1].

A simple cycle (or just a cycle) C of length t is a sequence $v_1v_2...v_tv_1$ of distinct vertices $v_1, ..., v_t$ with $v_iv_{i+1} \in E(G)$ for each $i \in \{1, ..., t\}$, where $v_{t+1} = v_1$. When $t = 2$, the cycle $C = v_1v_2v_1$ on two vertices v_1, v_2 coincides with the edge v_1v_2 , and when $t = 1$, the cycle $C = v_1$ coincides with the vertex v_1 . So, by this standard definition, all vertices and edges in a graph can be considered as cycles of lengths 1 and 2, respectively. A graph G is hamiltonian if G contains a Hamilton cycle, i.e. a cycle of length n . If Q is a cycle then we use $|Q|$ to denote the length of Q , that is $|Q| = |V(Q)|$. We call Q a large cycle in a graph G if it dominates some certain subgraph structures in G in a sense that every such structure has a vertex in common with Q . If Q dominates all vertices in G then clearly C is a Hamilton cycle. A cycle Q is a dominating cycle if it dominates all edges in G . A cycle Q is a PD_λ (path dominating) cycle if it dominates all paths in G of length at least some fixed integer λ . Finally, a cycle Q is a QD_λ (cycle dominating) cycle if it dominates all cycles in G of length at least λ .

We use n, q, δ and α to denote the order (the number of vertices), the size (the number of edges), minimum degree and independence number of a graph.

The connectivity κ is defined to be the minimum number of vertices whose removal disconnects G or reduces it to a single vertex K_1 . Let $s(G)$ denote the number of components of a graph G . A graph G is t -tough if $|S| \geq ts(G \setminus S)$ for every subset S of the vertex set $V(G)$ with $s(G \setminus S) > 1$. The toughness of G , denoted $\tau(G)$, is the maximum value of t for which G is t -tough (taking $\tau(K_n) = \infty$ for all $n \geq 1$). For a given longest cycle C we denote by \bar{p} and \bar{c} the lengths of a longest path and a longest cycle in $G \setminus C$.

Let a, b, t, k be integers with $k \leq t$. We use $H(a, b, t, k)$ to denote the graph obtained from $tK_a + \bar{K}_t$ by taking any k vertices in subgraph \bar{K}_t and joining each of them to all vertices of K_b . Let L_δ be the graph obtained from $3K_\delta + K_1$ by taking one vertex in each of three copies of K_δ and joining them each to other. For odd $n \geq 15$, construct the graph G_n from $\bar{K}_{\frac{n-1}{2}} + K_\delta + K_{\frac{n+1}{2}-\delta}$, where $n/3 \leq \delta \leq (n-5)/2$, by joining every vertex in K_δ to all other vertices and by adding a matching between all vertices in $K_{\frac{n+1}{2}-\delta}$ and $(n+1)/2 - \delta$ vertices in $\bar{K}_{\frac{n-1}{2}}$. It is easily seen that G_n is 1-tough but not hamiltonian. A variation of the graph G_n , with K_δ replaced by \bar{K}_δ and $\delta = (n-5)/2$, will be denoted by G_n^* .

3 Fundamentals and possible developments

The earliest two theoretical results in hamiltonian graph theory were developed in 1952 due to Dirac [3] in forms of a sufficient condition for a graph to be hamiltonian and a lower bound for the circumference c - the length of a longest cycle in a graph, based on order n and minimum degree δ .

Theorem A (Dirac [3], 1952). Every graph with $\delta \geq \frac{1}{2}n$ is hamiltonian.

Theorem B (Dirac [3], 1952). In every 2-connected graph, $c \geq \min\{n, 2\delta\}$.

The exact analogs of Theorems A and B for dominating cycles was given by Nash-Williams [8] and Jung [5].

Theorem C (Nash-Williams [8], 1971). Let G be a 2-connected graph with $\delta \geq \frac{1}{3}(n+2)$. Then each longest cycle in G is a dominating cycle.

Theorem D (Jung [5], 1981). Let G be a 3-connected graph. Then either $c \geq 3\delta - 3$ or each longest cycle in G is a dominating cycle.

Theorems C and D can be easily modified to the following stronger results under additional relation $\delta \geq \alpha$ between the minimum degree δ and independence number α .

Theorem E (Nash-Williams [8], 1971). Every 2-connected graph with $\delta \geq \alpha$ and $\delta \geq \frac{1}{3}(n+2)$ is hamiltonian.

Theorem F (Jung [5], 1981). In every 3-connected graph with $\delta \geq \alpha$, $c \geq \min \{n, 3\delta - 3\}$.

In 1972, Chvátal and Erdős [2] found a simple relation between connectivity κ and independence number α insuring the existence of a Hamilton cycle.

Theorem G (Chvátal and Erdős [2], 1972). Every graph with $\kappa \geq \alpha$ is hamiltonian.

One can easily observe that the most of results in hamiltonian graph theory are inspired by fundamental Theorems A-G by introducing various additional new ideas, generalizations, extensions, restrictions and structural limitations such as:

- **generalized and extended graph invariants** - degree sequences (Pósa type, Chvátal type), degree sums (Ore type, Fun type), neighborhood unions, generalized degrees, local connectivity, and so on,
- **extended list of path and cycle structures** - Hamilton, longest and dominating cycles, generalized cycles including Hamilton and dominating cycles as special cases, 2-factor, multiple Hamilton cycles, edge disjoint Hamilton cycles, powers of Hamilton cycles, k -ordered Hamilton cycles, arbitrary cycles, cycle systems, pancyclic-type cycle systems, cycles containing specified sets of vertices or edges, shortest cycles, analogous path structures, and so on,
- **structural (descriptive) limitations** - regular, planar, bipartite, chordal and interval graphs, graphs with forbidden subgraphs, Boolean graphs, hypercubes, and so on,
- **graph extensions** - hypergraphs, digraphs and orgraphs, labeled and weighted graphs, infinite graphs, random graphs, and so on.

Returning to Theorems A-F, observe that the bounds

$$\frac{n}{2}, \quad \frac{n+2}{3}, \quad 2\delta, \quad 3\delta - 3$$

in these theorems are the maximum that can be press out from graph invariants n , δ , α and their possible combinations. In other words, the impact of graph invariants n , δ and α on cycle structures is limited due to their nature.

A natural problem arises to find the exact analogs of fundamental Theorems A-F with alternative compositions of parameters by incorporating new advanced graph invariants such as connectivity κ and toughness τ - the best known graph invariants having much more impact on cycle structures in graphs.

4 The main contribution

In Theorems 1-3 we give three lower bounds for the length of a longest cycle C of a graph G in terms of minimum degree δ , connectivity κ and parameters \overline{p} , \overline{c} - the lengths of a longest path and longest cycle in $G \setminus C$, respectively. These bounds have no analogs in the area involving \overline{p} and \overline{c} as parameters, despite the fact that the idea of using $G \setminus C$ appropriate structures lies in the base of almost all existing proof techniques in trying to construct longer cycles in graphs by the following standard procedure: choose an initial cycle C_0 in G and try to enlarge it by replacing a segment P' of C_0 with a suitable path P'' longer than P' , having the same end vertices and passing through $G \setminus C_0$. To find suitable P' and P'' , one can use the paths or cycles (preferably large) in $G \setminus C_0$ and connections (preferably high) between these paths (cycles) and C_0 . The latter are closely related to \overline{p} , \overline{c} , as well as minimum degree δ (local connections) and connectivity κ (global connections).

We give also a number of exact analogs of fundamental Theorems A-F, concentrated especially on connectivity parameter κ .

The main contribution can be formulated as follows.

- Eighteen exact analogs of fundamental Theorems A-F were developed (Theorems 1-18) providing alternative advanced compositions of graph invariants.
- For fixed longest cycle C in a graph G , the appropriate $G \setminus C$ structures are intensively studied toward using these structures to construct longer cycles in graphs. The lengths \overline{p} and \overline{c} of a longest path and a longest cycle in $G \setminus C$, respectively, are incorporated into three lower bounds for $|C|$ as parameters (Theorems 1-3, having no analogs in the area), essentially improving the previous bounds.
- The connectivity invariant κ (the most interesting parameter related to cycle structures in graphs) appears as a parameter in the fundamental Theorem G, as well as in a number of exact analogs (Theorems 3-10) of the fundamental Theorems A-F in the following chronological order: 1972 (Chvátal and Erdős [2]), 1981a (Nikoghosyan [10]), 1981b (Nikoghosyan [10]), 1985a (Nikoghosyan [11]), 1985b (Nikoghosyan [12]), 2000 (Nikoghosyan [15]), 2005 (Lu, Liu, Tian [7]), 2009 (Nikoghosyan [17]), 2009a (Yamashita [23]), 2009b (Yamashita [23]), 2011a (Nikoghosyan [9]), 2011b (Nikoghosyan [9]).

5 Results

We begin with two lower bounds for the length of a longest cycle C in a graph G based on parameters \overline{p} and \overline{c} - the lengths of a longest path and a longest cycle in $G \setminus C$, respectively.

Theorem 1 [13] (1998)

Let G be a graph and C a longest cycle in G . Then

$$|C| \geq (\bar{p} + 2)(\delta - \bar{p}).$$

Example for sharpness: $(\kappa + 1)K_{\delta - \kappa + 1} + K_\kappa$.

Theorem 2 [14] (2000)

Let G be a graph and C a longest cycle in G . Then

$$|C| \geq (\bar{c} + 1)(\delta - \bar{c} + 1).$$

Example for sharpness: $(\kappa + 1)K_{\delta - \kappa + 1} + K_\kappa$.

For fixed δ , the bound in Theorem 2 (for example) is a strictly increasing function of \bar{c} when $0 \leq \bar{c} \leq \delta/2$, and is a strictly decreasing function when $\delta/2 \leq \bar{c} \leq \delta$ with global maximum

$$([\delta/2] + 1)(\delta/2 + 1) \cong \frac{(\delta + 2)^2}{4}$$

at $\bar{c} = [\delta/2]$.

My best result is the following lower bound for the circumference, providing increasing function on \bar{c} , κ and δ .

Theorem 3 [15] (2000)

Let G be a graph with $\kappa \geq 2$ and C a longest cycle in G . If $\bar{c} \geq \kappa$ then

$$|C| \geq \frac{(\bar{c} + 1)\kappa}{\bar{c} + \kappa + 1}(\delta + 2).$$

Otherwise,

$$|C| \geq \frac{(\bar{c} + 1)\bar{c}}{2\bar{c} + 1}(\delta + 2).$$

Example for sharpness: $(\kappa + 1)K_{\delta - \kappa + 1} + K_\kappa$.

Theorem 4 [10] (1981) (Analog of Theorem A)

Every 2-connected graph with

$$\delta \geq \frac{n + \kappa}{3}$$

is hamiltonian.

Examples for sharpness: $2K_\delta + K_1$; $H(1, \delta - \kappa + 1, \delta, \kappa)$ ($2 \leq \kappa < n/2$).

A short proof of Theorem 4 was given by Häggkvist [4].

Theorem 5 [11] (1985) (Analog of Theorem E)

Every 3-connected graph with

$$\delta \geq \max \left\{ \frac{n+2\kappa}{4}, \alpha \right\}$$

is hamiltonian.

Examples for sharpness: $3K_2 + K_2$; $4K_2 + K_3$, $H(1, 2, \kappa + 1, \kappa)$.

The graph $4K_2 + K_3$ shows that for $\kappa = 3$ the minimum degree bound $(n + 2\kappa)/4$ in Theorem 5 can not be replaced by $(n + 2\kappa - 1)/4$.

Theorem 6 [10] (1981) (Analog of Theorem B)
In every 3-connected graph,

$$c \geq \min\{n, 3\delta - \kappa\}.$$

Examples for sharpness: $3K_{\delta-1} + K_2$; $H(1, \delta - \kappa + 1, \delta, \kappa)$.

Theorem 7 [12] (1985) (Analog of Theorem F)
In every 4-connected graph with $\delta \geq \alpha$,

$$c \geq \min\{n, 4\delta - 2\kappa\}.$$

Examples for sharpness: $4K_2 + K_3$; $H(1, n - 2\delta, \delta, \kappa)$; $5K_2 + K_4$. The bound $4\delta - 2\kappa$ in Theorem 7 is sharp for $\kappa = 4$.

Theorem 8 [9] (2011) (Analog of Theorem F)
In every 4-connected graph with $\delta \geq \alpha$,

$$c \geq \min\{n, 4\delta - \kappa - 4\}.$$

Examples for sharpness: $4K_{\delta-2} + K_3$; $H(1, 2, \kappa + 1, \kappa)$; $H(2, n - 3\delta + 3, \delta - 1, \kappa)$.

Theorem 9 [17] (2011) (Analog of Theorem D)
If G is a 4-connected graph then either

$$c \geq 4\delta - 2\kappa$$

or G has a dominating cycle.

Examples for sharpness: $4K_2 + K_3$; $5K_2 + K_4$; $H(1, n - 2\delta, \delta, \kappa)$. Theorem 9 is sharp only for $\kappa = 4$ as can be seen from $5K_2 + K_4$.

Theorem 10 [9] (2011) (Analog of Theorem D)
If G is a 4-connected graph then either

$$c \geq 4\delta - \kappa - 4$$

or each longest cycle in G is a dominating cycle.

Examples for sharpness: $4K_{\delta-2} + K_3$; $H(2, \delta - \kappa + 1, \delta - 1, \kappa)$; $H(1, 2, \kappa + 1, \kappa)$.

Theorem 11 [16] (2009) (Generalized analog of Theorems A and C)
Let G be a graph, λ a positive integer and

$$\delta \geq \frac{n+2}{\lambda+1} + \lambda - 2.$$

If $\kappa \geq \lambda$ then each longest cycle in G is a $CD_{\min\{\lambda, \delta - \lambda + 1\}}$ -cycle.

Examples for sharpness: $\lambda K_{\lambda+1} + K_{\lambda-1}$ ($\lambda \geq 2$); $(\lambda + 1)K_{\delta - \lambda + 1} + K_\lambda$ ($\lambda \geq 1$); $H(\lambda - 1, \lambda, \lambda + 2, \lambda + 1)$ ($\lambda \geq 2$).

Theorem 12 [16] (2009) (Generalized analog of Theorems B and D)
Let G be a graph and λ a positive integer. If $\kappa \geq \lambda + 1$ then either

$$c \geq (\lambda + 1)(\delta - \lambda + 1)$$

or each longest cycle in G is a $CD_{\min\{\lambda, \delta - \lambda\}}$ -cycle.

Examples for sharpness: $(\lambda + 1)K_{\lambda+1} + K_\lambda$ ($\lambda \geq 1$); $(\lambda + 3)K_{\lambda-1} + K_{\lambda+2}$ ($\lambda \geq 2$); $(\lambda + 2)K_\lambda + K_{\lambda+1}$ ($\lambda \geq 1$).

Theorem 13 [16] (2009) (Generalized analog of Theorem F)
Let G be a graph and λ a positive integer. If $\kappa \geq \lambda + 2$ and $\delta \geq \alpha + \lambda - 1$ then

$$c \geq \min\{n, (\lambda + 2)(\delta - \lambda)\}.$$

Examples for sharpness: $(\lambda + 2)K_{\lambda+2} + K_{\lambda+1}$; $(\lambda + 4)K_\lambda + K_{\lambda+3}$; $(\lambda + 3)K_{\lambda+1} + K_{\lambda+2}$.

Theorem 14 [18] (2011) (Analog of Theorem A)
Every graph with $q \leq \delta^2 + \delta - 1$ is hamiltonian.

Examples for sharpness: The bound $\delta^2 + \delta - 1$ in Theorem 14 can not be relaxed to $\delta^2 + \delta$ since the graph $K_1 + 2K_\delta$ consisting of two copies of $K_{\delta+1}$ and having exactly one vertex in common, has $\delta^2 + \delta$ edges and is not hamiltonian.

Theorem 15 [19] (2011) (Analog of Theorem C)
Let G be a 2-connected graph. If

$$q \leq \begin{cases} 8 & \text{when } \delta = 2, \\ \frac{3(\delta-1)(\delta+2)-1}{2} & \text{when } \delta \geq 3, \end{cases}$$

then each longest cycle in G is a dominating cycle.

Examples for sharpness: To show that Theorem 15 is sharp, suppose first that $\delta = 2$. The graph $K_1 + 2K_2$ shows that the connectivity condition $\kappa \geq 2$ in Theorem 15 can not be relaxed by replacing it with $\kappa \geq 1$. The graph with vertex set $\{v_1, v_2, \dots, v_8\}$ and edge set

$$\{v_1v_2, v_2v_3, v_3v_4, v_4v_5, v_5v_6, v_6v_1, v_1v_7, v_7v_8, v_8v_4\},$$

shows that the size bound $q \leq 8$ can not be relaxed by replacing it with $q \leq 9$. Finally, the graph $K_2 + 3K_1$ shows that the conclusion "each longest cycle in G is a dominating cycle" can not be strengthened by replacing it with " G is hamiltonian". Analogously, we can use $K_1 + 2K_\delta$, $K_2 + 3K_{\delta-1}$ and $K_\delta + (\delta + 1)K_1$, respectively, to show that Theorem 15 is sharp when $\delta \geq 3$. So, Theorem 15 is best possible in all respects.

Theorem 16 [22] (2012) (Analog of Theorem B)
Let G be a graph with $\tau > 1$. Then either

$$c \geq \min\{n, 2\delta + 5\}$$

or G is the Petersen graph.

Theorem 17 [20] (2012) (Analog of Theorem C)
Let G be a graph with $\tau > 1$. If

$$\delta \geq \frac{n-2}{3}$$

then each longest cycle in G is a dominating cycle.

Theorem 18 [21] (2012) (Analog of Theorem C)
Let G be a 1-tough graph. If

$$\delta \geq \frac{n-2}{3}$$

then each longest cycle in G is a dominating cycle unless G belongs to an easily specified class of graphs.

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